積分の計算《基本演習》　(NO.2)　解答例 1 枚目

1. 次の不定積分を求めよ。

(1) $\int \sqrt{x}\ dx$

（解）$\int \sqrt{x}\ dx = \int x^{\frac{1}{2}} dx = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C$

$= \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C = \frac{2}{3}x\sqrt{x} + C$ 〃

(2) $\int (4x+3)^5 dx$

（解）$\int (4x+3)^5 dx$

$= \frac{1}{4} \cdot \frac{1}{5+1}(4x+3)^{5+1} + C$

$= \frac{1}{4} \cdot \frac{1}{6}(4x+3)^6 + C = \frac{1}{24}(4x+3)^6 + C$ 〃

（別解）$t = 4x+3$ とおくと

$\frac{dt}{dx} = 4 \quad \therefore dt = 4dx \quad \therefore \frac{1}{4}dt = dx$

$\int (4x+3)^5 dx = \int t^5 \cdot \frac{1}{4} dt = \frac{1}{4}\int t^5 dt$

$= \frac{1}{4} \cdot \frac{1}{6} t^6 + C = \frac{1}{24}(4x+3)^6 + C$ 〃

(3) $\int \left(\frac{1}{\sqrt{x^2+2}} - \frac{1}{x^2+2} \right) dx$

（解）$\int \frac{1}{\sqrt{x^2+2}} dx - \int \frac{1}{x^2+(\sqrt{2})^2} dx$

$= \log\left| x + \sqrt{x^2+2} \right| - \frac{1}{\sqrt{2}}\ \mathrm{Tan}^{-1} \frac{x}{\sqrt{2}} + C$ 〃

(4) $\int \frac{x^2+1}{x^3+3x+1} dx$

（解）$t = x^3+3x+1$ とおくと　$\frac{dt}{dx} = 3x^2+3$

$\therefore dt = 3(x^2+1)dx \quad \therefore \frac{1}{3}dt = (x^2+1)dx$

与式 $= \int \frac{1}{x^3+3x+1} \cdot (x^2+1) dx$

$= \int \frac{1}{t} \cdot \frac{1}{3} dt = \frac{1}{3}\int \frac{1}{t} dt = \frac{1}{3}\log|t| + C$

$= \frac{1}{3}\log|x^3+3x+1| + C$ 〃

（別解）$\int \frac{x^2+1}{x^3+3x+1} dx = \frac{1}{3}\int \frac{3x^2+3}{x^3+3x+1} dx$

$= \frac{1}{3}\int \frac{(x^3+3x+1)'}{x^3+3x+1} dx$

$= \frac{1}{3}\log\left|x^3+3x+1\right| + C$ 〃

(5) $\int x\sin x dx$

（解）$\int \sin x dx = -\cos x$,　$\int \cos x dx = \sin x$

であるから、部分積分法によって

$\int x\sin x dx = x \cdot (-\cos x) - \int (x)' \cdot (-\cos x) dx$

$= -x\cos x - \int 1 \cdot (-\cos x) dx$

$= -x\cos x + \int \cos x dx = -x\cos x + \sin x + C$ 〃

2. 次の定積分の値を求めよ。

(1) $\int_1^2 \frac{1}{x^3} dx$

（解）$\int_1^2 \frac{1}{x^3} dx = \int_1^2 x^{-3} dx = \left[\frac{1}{-3+1} x^{-3+1} \right]_1^2$

$= \left[\frac{1}{-2} x^{-2} \right]_1^2 = -\frac{1}{2}\left[\frac{1}{x^2} \right]_1^2 = -\frac{1}{2}\left(\frac{1}{2^2} - \frac{1}{1^2} \right)$

$= -\frac{1}{2}\left(\frac{1}{4} - 1 \right) = -\frac{1}{2}\left(-\frac{3}{4} \right) = \frac{3}{8}$ 〃

(2) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\, dx$

（解）$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\, dx = \Big[-\cos x \Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\Big[\cos x \Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= -\left(\cos\frac{\pi}{3} - \cos\frac{\pi}{6} \right) = -\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}$ 〃

(3) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$

（解）与式 $= \int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{2^2-x^2}} dx = \left[\ \mathrm{Sin}^{-1}\frac{x}{2} \right]_{\sqrt{2}}^{\sqrt{3}}$

$= \mathrm{Sin}^{-1}\frac{\sqrt{3}}{2} - \mathrm{Sin}^{-1}\frac{\sqrt{2}}{2} = \mathrm{Sin}^{-1}\frac{\sqrt{3}}{2} - \mathrm{Sin}^{-1}\frac{1}{\sqrt{2}}$

$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{4}{12}\pi - \frac{3}{12}\pi = \frac{\pi}{12}$ 〃

(4) $\int_0^1 xe^{2x} dx$

（解）$\int e^{2x} dx = \frac{1}{2}e^{2x}$ であるから、部分積分法によって

$\int_0^1 xe^{2x} dx = \left[x \cdot \frac{1}{2}e^{2x} \right]_0^1 - \int_0^1 (x)' \cdot \frac{1}{2}e^{2x} dx$

$= \frac{1}{2}\left[xe^{2x} \right]_0^1 - \int_0^1 1 \cdot \frac{1}{2}e^{2x} dx$

$= \frac{1}{2}\left(e^2 - 0 \right) - \frac{1}{2}\int_0^1 e^{2x} dx = \frac{1}{2}e^2 - \frac{1}{2}\left[\frac{1}{2}e^{2x} \right]_0^1$

$= \frac{1}{2}e^2 - \frac{1}{4}\left[e^{2x} \right]_0^1 = \frac{1}{2}e^2 - \frac{1}{4}(e^2 - e^0)$

$= \frac{2}{4}e^2 - \frac{1}{4}(e^2-1) = \frac{1}{4}(2e^2 - e^2 + 1) = \frac{1}{4}(e^2+1)$ 〃

(5) $\displaystyle\int_{\sqrt{3}}^{2}\sqrt{4-x^2}dx$

（解）

$$与式 = \int_{\sqrt{3}}^{2}\sqrt{2^2-x^2}dx$$

$$= \frac{1}{2}\left[x\sqrt{2^2-x^2}+2^2\ \mathrm{Sin}^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2}\left[x\sqrt{4-x^2}+4\ \mathrm{Sin}^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2}\left\{\left(2\cdot 0+4\cdot\ \mathrm{Sin}^{-1}1\right)-\left(\sqrt{3}\cdot 1+4\cdot\ \mathrm{Sin}^{-1}\frac{\sqrt{3}}{2}\right)\right\}$$

$$= \frac{1}{2}\left\{\left(0+4\cdot\frac{\pi}{2}\right)-\left(\sqrt{3}+4\cdot\frac{\pi}{3}\right)\right\}$$

$$= \pi-\frac{\sqrt{3}}{2}-\frac{2}{3}\pi = \frac{\pi}{3}-\frac{\sqrt{3}}{2}\quad〃$$

(別解)

$$与式 = \int_{\sqrt{3}}^{2}\sqrt{2^2-x^2}dx$$

$x = 2\sin t$ とおくと

x	$\sqrt{3}$	$\to$	2
t	$\frac{\pi}{3}$	$\to$	$\frac{\pi}{2}$

$$\frac{dx}{dt} = 2\cos t \quad \therefore dx = 2\cos t dt$$

$$\sqrt{2^2-x^2} = \sqrt{2^2-(2\sin t)^2} = 2\sqrt{1-\sin^2 t}$$

$$= 2\sqrt{\cos^2 t} = 2|\cos t| = 2\cos t$$

$$与式 = \int_{\sqrt{3}}^{2}2\cos t\cdot 2\cos t dt = 4\int_{\sqrt{3}}^{2}\cos^2 t dt$$

$$= 4\int_{\sqrt{3}}^{2}\frac{1+\cos 2t}{2}dt$$

$$= 2\int_{\sqrt{3}}^{2}(1+\cos 2t)dt$$

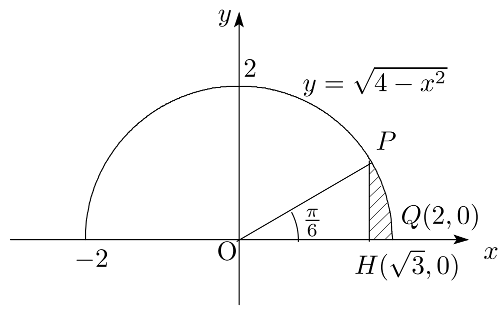
$$= 2\left[t+\frac{1}{2}\sin 2t\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= 2\left\{\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\left(\frac{\pi}{3}+\frac{1}{2}\sin\frac{2\pi}{3}\right)\right\}$$

$$= 2\left(\frac{\pi}{2}+\frac{1}{2}\cdot 0-\frac{\pi}{3}-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)$$

$$= \pi-2\cdot\frac{\pi}{3}-\frac{\sqrt{3}}{2} = \frac{\pi}{3}-\frac{\sqrt{3}}{2}\quad〃$$

《参考》



斜線の部分の面積を S とすると

$$S = 扇形\ OPQ-\triangle OPH$$

$$= \frac{1}{2}\cdot 2^2\cdot\frac{\pi}{6}-\frac{1}{2}\cdot\sqrt{3}\cdot 2\sin\frac{\pi}{6}$$

$$= \frac{1}{2}\cdot 4\cdot\frac{\pi}{6}-\frac{1}{2}\cdot\sqrt{3}\cdot 2\cdot\frac{1}{2} = \frac{\pi}{3}-\frac{\sqrt{3}}{2}$$

$$\int_{\sqrt{3}}^{2}\sqrt{4-x^2}dx = S = \frac{\pi}{3}-\frac{\sqrt{3}}{2}$$

《参考》

a が正の定数のとき,

次の公式を部分積分法によって導びけ.

$$\int\sqrt{a^2-x^2}dx = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\ \mathrm{Sin}^{-1}\frac{x}{a}\right)$$

(解)

$$I = \int\sqrt{a^2-x^2}dx = \int 1\cdot\sqrt{a^2-x^2}dx$$

$$= x\sqrt{a^2-x^2}-\int x\cdot\left(\sqrt{a^2-x^2}\right)'dx$$

$$= x\sqrt{a^2-x^2}-\int x\cdot\frac{-2x}{2\sqrt{a^2-x^2}}dx$$

$$= x\sqrt{a^2-x^2}-\int\frac{-x^2}{\sqrt{a^2-x^2}}dx$$

$$= x\sqrt{a^2-x^2}-\int\frac{a^2-x^2-a^2}{\sqrt{a^2-x^2}}dx$$

$$= x\sqrt{a^2-x^2}-\int\sqrt{a^2-x^2}dx+a^2\int\frac{1}{\sqrt{a^2-x^2}}dx$$

$$= x\sqrt{a^2-x^2}-I+a^2\ \mathrm{Sin}^{-1}\frac{x}{a}$$

$$\therefore 2I = x\sqrt{a^2-x^2}+a^2\ \mathrm{Sin}^{-1}\frac{x}{a}$$

$$\therefore I = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\ \mathrm{Sin}^{-1}\frac{x}{a}\right)$$

$$\therefore \int\sqrt{a^2-x^2}dx = \frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\ \mathrm{Sin}^{-1}\frac{x}{a}\right)$$

積分の計算《基本演習》 (NO.2) 解答例 2枚目

3. 次の不定積分を求めよ。

(1) $\int(\sqrt{x}-x^2)dx$

（解）与式 $=\int(x^{\frac{1}{2}}-x^2)dx$

$=\dfrac{1}{\frac{3}{2}}x^{\frac{3}{2}}-\dfrac{1}{3}x^3+C$

$=\dfrac{2}{3}\sqrt{x^3}-\dfrac{1}{3}x^3+C=\dfrac{2}{3}x\sqrt{x}-\dfrac{1}{3}x^3+C$ 〃

(2) $\int(2x-3)^6dx$

（解）$\int(2x-3)^6dx$

$=\dfrac{1}{2}\cdot\dfrac{1}{7}(2x-3)^7+C=\dfrac{1}{14}(2x-3)^7+C$ 〃

（別解）$t=2x-3$ とおくと

$\dfrac{dt}{dx}=2 \quad \therefore dt=2dx \quad \therefore \dfrac{1}{2}dt=dx$

$\int(2x-3)^6dx=\int t^6\cdot\dfrac{1}{2}dt=\dfrac{1}{2}\int t^6dt$

$=\dfrac{1}{2}\cdot\dfrac{1}{7}t^7+C=\dfrac{1}{14}(2x-3)^7+C$ 〃

(3) $\int\left(\dfrac{1}{x^2+9}-\dfrac{1}{\sqrt{x^2+9}}\right)dx$

（解）与式 $=\int\dfrac{1}{x^2+3^2}dx-\int\dfrac{1}{\sqrt{x^2+9}}dx$

$=\dfrac{1}{3}\ \mathrm{Tan}^{-1}\dfrac{x}{3}-\log\left|x+\sqrt{x^2+9}\right|+C$ 〃

(4) $\int\dfrac{x}{(x^2+1)^2}dx$

（解）$t=x^2+1$ とおくと $\dfrac{dt}{dx}=2x$

$\therefore dt=2xdx \quad \therefore \dfrac{1}{2}dt=xdx$

与式 $=\int\dfrac{1}{(x^2+1)^2}\cdot xdx=\int\dfrac{1}{t^2}\cdot\dfrac{1}{2}dt$

$=\dfrac{1}{2}\int t^{-2}dt=\dfrac{1}{2}\cdot\dfrac{1}{-2+1}t^{-2+1}+C$

$=-\dfrac{1}{2}\cdot t^{-1}+C=-\dfrac{1}{2t}+C=-\dfrac{1}{2(x^2+1)}+C$ 〃

（別解）$x=\tan\theta$ とおくと,

$\dfrac{dx}{d\theta}=\sec^2\theta \quad \therefore dx=\sec^2\theta d\theta$

$(x^2+1)^2=(\tan^2\theta+1)^2=(\sec^2\theta)^2$

$\int\dfrac{x}{(x^2+1)^2}dx=\int\dfrac{\tan\theta}{(\sec^2\theta)^2}\sec^2\theta d\theta$

$=\int\tan\theta\cdot\dfrac{1}{\sec^2\theta}d\theta$

$=\int\dfrac{\sin\theta}{\cos\theta}\cdot\cos^2\theta d\theta=\int\sin\theta\cos\theta d\theta$

$=\dfrac{1}{2}\int2\sin\theta\cos\theta d\theta=\dfrac{1}{2}\int\sin2\theta d\theta$

$=\dfrac{1}{2}\left(-\dfrac{1}{2}\cos2\theta\right)+c=-\dfrac{1}{4}(2\cos^2\theta-1)+c$

$=-\dfrac{1}{2}\left(\dfrac{1}{\sec^2\theta}\right)+\dfrac{1}{4}+c=-\dfrac{1}{2(1+\tan^2\theta)}+\dfrac{1}{4}+c$

$=-\dfrac{1}{2(1+x^2)}+\dfrac{1}{4}+c$

$\dfrac{1}{4}+c=C$ とおくと、

与式 $=-\dfrac{1}{2(x^2+1)}+C$ 〃

(5) $\int x\cos xdx$

（解）$\int\cos xdx=\sin x$, $\int\sin xdx=-\cos x$

であるから、部分積分法によって

$\int x\cos xdx=x\cdot\sin x-\int(x)'\cdot\sin xdx$

$=x\sin x-\int\sin xdx$

$=x\sin x-(-\cos x)+C$

$=x\sin x+\cos x+C$ 〃

4. 次の定積分の値を求めよ。

(1) $\int_{-1}^{2}(x+2-x^2)dx$

（解）$\int_{-1}^{2}(x+2-x^2)dx$

$=\left[\dfrac{1}{2}x^2+2x-\dfrac{1}{3}x^3\right]_{-1}^{2}$

$=\left(2+4-\dfrac{8}{3}\right)-\left(\dfrac{1}{2}-2+\dfrac{1}{3}\right)=\dfrac{9}{2}$ 〃

（別解）与式 $=-\int_{-1}^{2}(x^2-x-2)dx$

$=-\int_{-1}^{2}(x+1)(x-2)dx=-\int_{-1}^{2}\{x-(-1)\}(x-2)dx$

$=-\left\{-\dfrac{1}{6}\left(2-(-1)\right)^3\right\}=\dfrac{1}{6}\cdot3^3=\dfrac{9}{2}$ 〃

(2) $\int_{0}^{\frac{\pi}{4}}(\cos x-\sin x)dx$

（解）$\int_{0}^{\frac{\pi}{4}}(\cos x-\sin x)dx$

$=\left[\sin x-(-\cos x)\right]_{0}^{\frac{\pi}{4}}=\left[\sin x+\cos x\right]_{0}^{\frac{\pi}{4}}$

$=\left(\sin\dfrac{\pi}{4}+\cos\dfrac{\pi}{4}\right)-(\sin0+\cos0)$

$=\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}\right)-(0+1)=\sqrt{2}-1$ 〃

(3) $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}dx$

（解）与式 $\displaystyle=\int_0^3 \frac{1}{\sqrt{3^2-x^2}}dx=\Big[\ \mathrm{Sin}^{-1}\frac{x}{3}\Big]_0^3$

$\displaystyle=\mathrm{Sin}^{-1}1-\mathrm{Sin}^{-1}0=\frac{\pi}{2}-0=\frac{\pi}{2}$ 〃

（別解）与式 $\displaystyle=\int_0^3 \frac{1}{\sqrt{3^2-x^2}}dx$

＜計算＞	
$x=0$ のとき	$x=3$ のとき
$0=3\sin\theta$	$3=3\sin\theta$
$\sin\theta=0$	$\sin\theta=1$
$\theta=0$	$\theta=\frac{\pi}{2}$

$x=3\sin\theta\left(0\leqq\theta\leqq\dfrac{\pi}{2}\right)$ とおくと

$\dfrac{dx}{d\theta}=3\cos\theta \quad \therefore dx=3\cos\theta d\theta$

x	$0\ \to\ 3$
θ	$0\ \to\ \frac{\pi}{2}$

$\sqrt{3^2-x^2}=\sqrt{9-9\sin^2\theta}=\sqrt{9(1-\sin^2\theta)}$

$=\sqrt{9\cos^2\theta}=\sqrt{(3\cos\theta)^2}=|3\cos\theta|=3\cos\theta$

与式 $\displaystyle=\int_0^{\frac{\pi}{2}}\frac{1}{3\cos\theta}\cdot 3\cos\theta d\theta$

$\displaystyle=\int_0^{\frac{\pi}{2}}1\,d\theta=\Big[\ \theta\ \Big]_0^{\frac{\pi}{2}}=\frac{\pi}{2}$ 〃

(4) $\displaystyle\int_0^1 xe^{3x}dx$

（解）$\displaystyle\int e^{3x}dx=\frac{1}{3}e^{3x}$

であるから、 部分積分法によって

$\displaystyle\int_0^1 xe^{3x}dx=\Big[x\cdot\frac{1}{3}e^{3x}\Big]_0^1-\int_0^1(x)'\cdot\frac{1}{3}e^{3x}dx$

$\displaystyle=\frac{1}{3}\Big[xe^{3x}\Big]_0^1-\int_0^1 1\cdot\frac{1}{3}e^{3x}dx$

$\displaystyle=\frac{1}{3}\left(e^3-0\right)-\frac{1}{3}\int_0^1 e^{3x}dx=\frac{1}{3}e^3-\frac{1}{3}\Big[\frac{1}{3}e^{3x}\Big]_0^1$

$\displaystyle=\frac{1}{3}e^3-\frac{1}{9}\Big[e^{3x}\Big]_0^1=\frac{1}{3}e^3-\frac{1}{9}(e^3-e^0)$

$\displaystyle=\frac{3}{9}e^3-\frac{1}{9}(e^3-1)=\frac{1}{9}(2e^3+1)$ 〃

（別解）$\displaystyle\int e^{3x}dx=\frac{1}{3}e^{3x}$ であるから,

部分積分法によって

$\displaystyle\int xe^{3x}dx=x\cdot\frac{1}{3}e^{3x}-\int(x)'\cdot\frac{1}{3}e^{3x}dx$

$\displaystyle=\frac{1}{3}xe^{3x}-\int 1\cdot\frac{1}{3}e^{3x}dx=\frac{1}{3}xe^{3x}-\frac{1}{3}\int e^{3x}dx$

$\displaystyle=\frac{1}{3}xe^{3x}-\frac{1}{3}\cdot\frac{1}{3}e^{3x}=\frac{1}{9}(3x-1)e^{3x}$

$\displaystyle\therefore\int_0^1 xe^{3x}dx=\frac{1}{9}\Big[(3x-1)e^{3x}\Big]_0^1$

$\displaystyle=\frac{1}{9}\left\{2\cdot e^3-(-1)\cdot e^0\right\}=\frac{1}{9}(2e^3+1)$ 〃

《注意》(別解) では積分定数は省略してある。

≪参考≫定積分の値を求める場合、
(別解) のように予め不定積分を求めて、
その後で定積分の値を求めてもよい。

(5) $\displaystyle\int_0^4\sqrt{16-x^2}dx$

（解）与式 $\displaystyle=\int_0^4\sqrt{4^2-x^2}dx$

$\displaystyle=\frac{1}{2}\Big[x\sqrt{4^2-x^2}+4^2\ \mathrm{Sin}^{-1}\frac{x}{4}\Big]_0^4$

$\displaystyle=\frac{1}{2}\Big\{\Big(4\cdot\sqrt{0}+16\cdot\mathrm{Sin}^{-1}1\Big)-\Big(0\cdot\sqrt{16}+16\cdot\mathrm{Sin}^{-1}0\Big)\Big\}$

$\displaystyle=\frac{1}{2}\cdot 16\cdot\ \mathrm{Sin}^{-1}1=\frac{1}{2}\cdot 16\cdot\frac{\pi}{2}=4\pi$ 〃

（別解）与式 $\displaystyle=\int_0^4\sqrt{4^2-x^2}dx$

＜計算＞	
$x=0$ のとき	$x=4$ のとき
$0=4\sin\theta$	$4=4\sin\theta$
$\sin\theta=0$	$\sin\theta=1$
$\theta=0$	$\theta=\frac{\pi}{2}$

$x=4\sin\theta\left(0\leqq\theta\leqq\dfrac{\pi}{2}\right)$ とおくと

$\dfrac{dx}{d\theta}=4\cos\theta \quad \therefore dx=4\cos\theta d\theta$

x	$0\ \to\ 4$
θ	$0\ \to\ \frac{\pi}{2}$

$\sqrt{4^2-x^2}=\sqrt{16-16\sin^2\theta}=\sqrt{16(1-\sin^2\theta)}$

$=\sqrt{16\cos^2\theta}=\sqrt{(4\cos\theta)^2}=|4\cos\theta|=4\cos\theta$

与式 $\displaystyle=\int_0^{\frac{\pi}{2}}4\cos\theta\cdot 4\cos\theta d\theta$

$\displaystyle=16\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta=16\int_0^{\frac{\pi}{2}}\frac{1+\cos 2\theta}{2}d\theta$

$\displaystyle=8\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)d\theta=8\Big[\theta+\frac{1}{2}\sin 2\theta\Big]_0^{\frac{\pi}{2}}$

$\displaystyle=8\left\{\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\left(0+\frac{1}{2}\sin 0\right)\right\}$

$\displaystyle=8\cdot\frac{\pi}{2}=4\pi$ 〃

≪参考≫ 中心が原点で、半径 4 の円 $x^2+y^2=4^2$ の面積の $\dfrac{1}{4}$ であるから

与式 $\displaystyle=\int_0^4\sqrt{4^2-x^2}dx$

$\displaystyle=\frac{1}{4}\times\pi\cdot 4^2=4\pi$ 〃