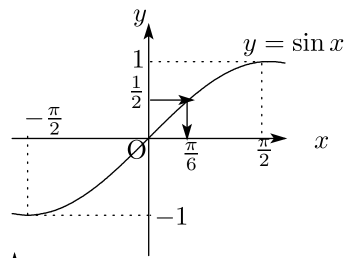
微分法 ≪基礎≫ 小テスト　$(No.7)$　解答例

1. 次の値を求めよ。

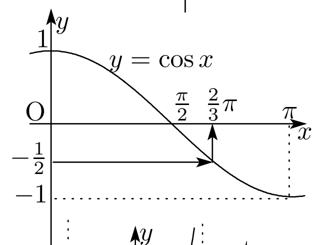
(1)　$\mathrm{Sin}^{-1}\dfrac{1}{2}$



(解) $\sin\dfrac{\pi}{6}=\dfrac{1}{2}$

$\therefore\ \mathrm{Sin}^{-1}\dfrac{1}{2}=\dfrac{\pi}{6}$　〃

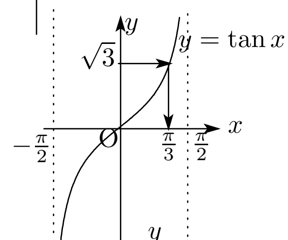
(2)　$\mathrm{Cos}^{-1}\left(-\dfrac{1}{2}\right)$



(解) $\cos\dfrac{2}{3}\pi=-\dfrac{1}{2}$

$\therefore\ \mathrm{Cos}^{-1}\left(-\dfrac{1}{2}\right)=\dfrac{2}{3}\pi$　〃

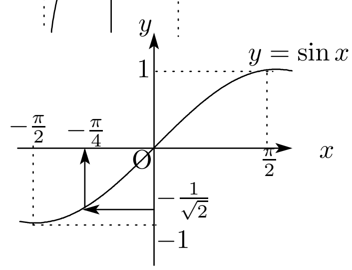
(3)　$\mathrm{Tan}^{-1}\sqrt{3}$



(解) $\tan\dfrac{\pi}{3}=\sqrt{3}$

$\therefore\ \mathrm{Tan}^{-1}\sqrt{3}=\dfrac{\pi}{3}$　〃

(4)　$\mathrm{Sin}^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$



(解) $\sin\left(-\dfrac{\pi}{4}\right)=-\dfrac{1}{\sqrt{2}}$

$\therefore\ \mathrm{Sin}^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)=-\dfrac{\pi}{4}$ 〃

2. 次の関数を微分せよ。

(1)　$y=\mathrm{Sin}^{-1}\dfrac{x}{3}$

(解) $u=\dfrac{x}{3}$ とおくと　$y=\mathrm{Sin}^{-1}u$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}\ \mathrm{Sin}^{-1}u\cdot\frac{d}{dx}\frac{x}{3}=\frac{1}{\sqrt{1-u^2}}\cdot\frac{1}{3}=\frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\cdot\frac{1}{3}=\frac{1}{\sqrt{9-x^2}}\quad〃$$

(2)　$y=\mathrm{Cos}^{-1}\dfrac{1}{x}$　　$(x>1)$

（解）$u=\dfrac{1}{x}$ とおくと　$y=\mathrm{Cos}^{-1}u$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}\ \mathrm{Cos}^{-1}u\cdot\frac{d}{dx}\frac{1}{x}=-\frac{1}{\sqrt{1-u^2}}\cdot\left(-\frac{1}{x^2}\right)=\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot\frac{1}{x}\cdot\frac{1}{x}$$

$$=\frac{1}{\sqrt{\left(1-\frac{1}{x^2}\right)\cdot x^2}}\cdot\frac{1}{x}=\frac{1}{x\sqrt{x^2-1}}\quad〃$$

(3)　$y=\mathrm{Tan}^{-1}\sqrt{x}$　　$(x>0)$

（解）$u=\sqrt{x}$ とおくと　$y=\mathrm{Tan}^{-1}u$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}\ \mathrm{Tan}^{-1}u\cdot\frac{d}{dx}\sqrt{x}=\frac{1}{1+u^2}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2(1+x)\sqrt{x}}\quad〃$$

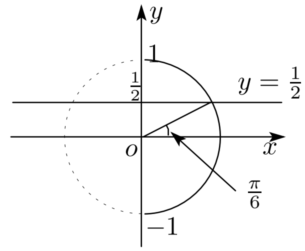
≪計算≫ $y=\sqrt{x}=x^{\frac{1}{2}}$ を微分すると

$$y'=\frac{1}{2}\cdot x^{\frac{1}{2}-1}=\frac{1}{2}\cdot x^{-\frac{1}{2}}=\frac{1}{2}\cdot\frac{1}{x^{\frac{1}{2}}}=\frac{1}{2}\cdot\frac{1}{\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

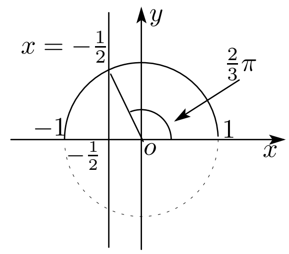
≪別解の研究≫

1. 次の値を求めよ。

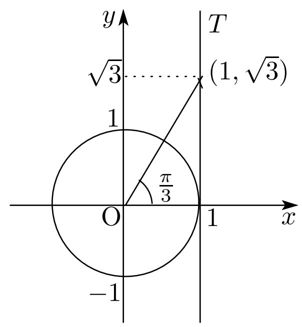
(1)　$\mathrm{Sin}^{-1}\,\dfrac{1}{2}=\dfrac{\pi}{6}$　〃



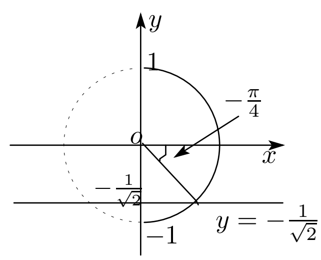
(2)　$\mathrm{Cos}^{-1}\left(-\dfrac{1}{2}\right)=\dfrac{2}{3}\pi$　〃



(3)　$\mathrm{Tan}^{-1}\sqrt{3}=\dfrac{\pi}{3}$　〃



(4)　$\mathrm{Sin}^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)=-\dfrac{\pi}{4}$　〃



2. 次の関数を微分せよ。

(1)　$y=\mathrm{Sin}^{-1}\,\dfrac{x}{3}$

(解) $y'=\dfrac{1}{\sqrt{1-(\frac{x}{3})^2}}\cdot\left(\dfrac{x}{3}\right)'=\dfrac{1}{\sqrt{1-\frac{x^2}{9}}}\cdot\dfrac{1}{3}=\dfrac{1}{\sqrt{(1-\frac{x^2}{9})\cdot 9}}=\dfrac{1}{\sqrt{9-x^2}}$　〃

(2)　$y=\mathrm{Cos}^{-1}\,\dfrac{1}{x}$　　$(x>1)$

(解) $y'=-\dfrac{1}{\sqrt{1-(\frac{1}{x})^2}}\cdot\left(\dfrac{1}{x}\right)'=-\dfrac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\left(-\dfrac{1}{x^2}\right)=\dfrac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\dfrac{1}{x^2}$

$=\dfrac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\dfrac{1}{x}\cdot\dfrac{1}{x}=\dfrac{1}{\sqrt{(1-\frac{1}{x^2})\cdot x^2}}\cdot\dfrac{1}{x}=\dfrac{1}{x\sqrt{x^2-1}}$　〃

≪計算 1≫ $y=\dfrac{1}{x}=x^{-1}$ を微分すると　　$y'=-1\cdot x^{-1-1}=-x^{-2}=-\dfrac{1}{x^2}$

≪計算 2≫ $y=\dfrac{1}{x}$ を商の微分法で微分すると　　$y'=-\dfrac{(x)'}{x^2}=-\dfrac{1}{x^2}$

(3)　$y=\mathrm{Tan}^{-1}\sqrt{x}$　　$(x>0)$

(解) $y'=\dfrac{1}{1+(\sqrt{x})^2}\cdot(\sqrt{x})'=\dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}}=\dfrac{1}{2(1+x)\sqrt{x}}$　〃