微分法 ≪基礎≫ 小テスト　$(No.8)$　解答例

1. 次の極限値を求めよ。

(1) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^x$

(解) $3x=h$ とおくと　$x=\dfrac{h}{3}$

$x\to\infty$ のとき　$h\to\infty$

$$\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^x=\lim_{h\to\infty}\left(1+\frac{1}{h}\right)^{\frac{h}{3}}=\lim_{h\to\infty}\left\{\left(1+\frac{1}{h}\right)^h\right\}^{\frac{1}{3}}=e^{\frac{1}{3}}=\sqrt[3]{e}\quad〃$$

(2) $\displaystyle\lim_{h\to 0}(1-2h)^{\frac{1}{h}}$

（解）$-2h=x$ とおくと　$h=-\dfrac{x}{2}$　$\therefore\dfrac{1}{h}=-\dfrac{2}{x}$

$h\to 0$ のとき　$x\to 0$

$$\lim_{h\to 0}(1-2h)^{\frac{1}{h}}=\lim_{x\to 0}(1+x)^{-\frac{2}{x}}=\lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{-2}=e^{-2}=\frac{1}{e^2}\quad〃$$

2. 次の関数を微分せよ。

(1) $y=\log|x^2-1|$

(解 1) $u=x^2-1$ とおくと　$y=\log|u|$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}\log|u|\cdot\frac{d}{dx}(x^2-1)=\frac{1}{u}\cdot(2x)=\frac{1}{x^2-1}\cdot 2x=\frac{2x}{x^2-1}\quad〃$$

（解 2）$y'=\dfrac{1}{x^2-1}\cdot(x^2-1)'=\dfrac{1}{x^2-1}\cdot(2x)=\dfrac{2x}{x^2-1}$　〃

(2) $y=e^{x^3}$

(解 1) $u=x^3$ とおくと　$y=e^u$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{d}{du}e^u\cdot\frac{d}{dx}(x^3)=e^u\cdot(3x^2)=e^{x^3}\cdot 3x^2=3x^2e^{x^3}\quad〃$$

(解 2) $y'=e^{x^3}\cdot(x^3)'=e^{x^3}\cdot(3x^2)=3x^2e^{x^3}$　〃

3. 対数微分法によって、次の関数の導関数を求めよ。

$y=x^{2x}$　　$(x>0)$

(解)　両辺の対数をとると　　$\log y=\log x^{2x}$

$\therefore\log y=2x\log x$

両辺をで x 微分すると

$\dfrac{d}{dx}(\log y)=\dfrac{d}{x}(2x\log x)$

$\dfrac{d}{dy}(\log y)\cdot\dfrac{dy}{dx}=(2x)'\cdot\log x+2x\cdot(\log x)'$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx}=2\cdot\log x+2x\cdot\dfrac{1}{x}$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx}=2\log x+2$

$\therefore\ \dfrac{dy}{dx}=y\cdot(2\log x+2)=x^{2x}\cdot 2(\log x+1)=2x^{2x}(\log x+1)$　〃

≪参考資料≫

1. 次の関数を微分せよ。

(1) $y = x^{\sqrt{5}}$

(解) $y' = \sqrt{5} \cdot x^{\sqrt{5}-1}$ 〃

(2) $y = x^2 \log x$

(解) $y' = (x^2)' \cdot \log x + x^2 \cdot (\log x)' = 2x \cdot \log x + x^2 \cdot \frac{1}{x} = 2x \log x + x = x(2\log x + 1)$ 〃

(3) $y = \log \sqrt{|\cos x|}$

(解 1) $y = \log \sqrt{|\cos x|} = \log(|\cos x|)^{\frac{1}{2}} = \dfrac{1}{2} \log |\cos x|$

$u = \cos x$ とおくと　$y = \dfrac{1}{2} \log |u|$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{d}{du}\left(\frac{1}{2}\log|u|\right) \cdot \frac{d}{dx}(\cos x) = \frac{1}{2} \cdot \frac{1}{u} \cdot (-\sin x)$$

$$= \frac{1}{2} \cdot \frac{1}{\cos x} \cdot (-\sin x) = -\frac{1}{2}\,\frac{\sin x}{\cos x} = -\frac{1}{2}\,\tan x$$ 〃

(解 2) $y = \log \sqrt{|\cos x|} = \log(|\cos x|)^{\frac{1}{2}} = \dfrac{1}{2} \log |\cos x|$

$$y' = \frac{1}{2} \cdot \frac{1}{\cos x} \cdot (\cos x)' = \frac{1}{2} \cdot \frac{1}{\cos x} \cdot (-\sin x) = -\frac{1}{2}\,\frac{\sin x}{\cos x} = -\frac{1}{2}\,\tan x$$ 〃

(4) $y = \log |\tan x|$

(解 1) $u = \tan x$ とおくと　$y = \log |u|$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{d}{du}\log|u| \cdot \frac{d}{dx}(\tan x) = \frac{1}{u} \cdot \sec^2 x = \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} = \frac{1}{\sin x \cos x}$$ 〃

(解 2) $y' = \dfrac{1}{\tan x} \cdot (\tan x)' = \dfrac{1}{\tan x} \cdot \sec^2 x = \dfrac{1}{\frac{\sin x}{\cos x}} \cdot \dfrac{1}{\cos^2 x} = \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos^2 x} = \dfrac{1}{\sin x \cos x}$ 〃

2. 次の関数を微分せよ。

(1) $y = 2^x$

(解) $y' = 2^x \log 2$ 〃

(2) $y = \log(e^x + e^{-x})$

(解) $y' = \dfrac{1}{e^x + e^{-x}} \cdot (e^x + e^{-x})' = \dfrac{1}{e^x + e^{-x}} \cdot \{e^x + e^{-x} \cdot (-x)'\}$

$$= \frac{1}{e^x + e^{-x}} \cdot \{e^x + e^{-x} \cdot (-1)\} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$ 〃

(3) $y = \dfrac{1}{3^x}$

(解 1)　商の微分法で微分すると

$$y' = -\frac{(3^x)'}{(3^x)^2} = -\frac{3^x \log 3}{(3^x)^2} = -\frac{\log 3}{3^x}$$ 〃

(解 2) $y = \dfrac{1}{3^x} = 3^{-x}$

$$y' = 3^{-x} \cdot (-x)' \cdot \log 3 = \frac{1}{3^x} \cdot (-1) \cdot \log 3 = -\frac{\log 3}{3^x}$$ 〃