第 3 章　線形代数《 § 2 行列と行列式 》

$\boxed{183}$ 次の連立方程式をガウスの消去法（掃き出し法）を用いて解け.

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

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[解] 与式をガウスの消去法で解く.

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & -1 & -1 & -3 \\ 2 & -2 & 1 & 1 \end{array}\right) \xrightarrow[3\text{行}-1\text{行}\times 2]{2\text{行}-1\text{行}\times 2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & 1 & -7 \\ 0 & -6 & 3 & -3 \end{array}\right)$$

$$\xrightarrow{2\text{行}-3\text{行}\times 1} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -2 & -4 \\ 0 & -6 & 3 & -3 \end{array}\right)$$

$$\xrightarrow[3\text{行}+2\text{行}\times 6]{1\text{行}-2\text{行}\times 2} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & -9 & -27 \end{array}\right)$$

$$\xrightarrow{3\text{行}\times\left(-\frac{1}{9}\right)} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\xrightarrow[2\text{行}+3\text{行}\times 2]{1\text{行}-3\text{行}\times 3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\therefore \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \therefore x = 1,\ y = 2,\ z = 3$$

《　検算　》$x = 1,\ y = 2,\ z = 3$ のとき,

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1\times 1 + 2\times 2 + (-1)\times 3 \\ 2\times 1 + (-1)\times 2 + (-1)\times 3 \\ 2\times 1 + (-2)\times 2 + 1\times 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

184 次の連立方程式を行列とベクトルを用いて書き直し, クラメルの公式を用いて解け.

$$\begin{cases} 2x - y + z &= 7 \\ x + 2y - 3z &= -1 \\ x - 3y - z &= -2 \end{cases}$$

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[解] 与式を行列とベクトルを用いて書き直すと,

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ -2 \end{pmatrix}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -3 & -1 \end{vmatrix} = \{(-4) + 3 + (-3)\} - \{2 + 18 + 1\} = -25$$

$$x = \frac{\begin{vmatrix} 7 & -1 & 1 \\ -1 & 2 & -3 \\ -2 & -3 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -3 & -1 \end{vmatrix}} = \frac{\{(-14) + (-6) + 3\} - \{(-4) + 63 + (-1)\}}{-25} = \frac{-75}{-25} = 3$$

$$y = \frac{\begin{vmatrix} 2 & 7 & 1 \\ 1 & -1 & -3 \\ 1 & -2 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -3 & -1 \end{vmatrix}} = \frac{\{2 + (-21) + (-2)\} - \{(-1) + 12 + (-7)\}}{-25} = \frac{-25}{-25} = 1$$

$$z = \frac{\begin{vmatrix} 2 & -1 & 7 \\ 1 & 2 & -1 \\ 1 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -3 & -1 \end{vmatrix}} = \frac{\{(-8) + 1 + (-21)\} - \{14 + 6 + 2\}}{-25} = \frac{-50}{-25} = 2$$

$\therefore\ x = 3,\ y = 1,\ z = 2$

《 検算 》 $x = 3,\ y = 1,\ z = 2$ のとき,

$$\begin{cases} 2x - y + z &= 2 \times 3 - 1 + 2 &= 7 \\ x + 2y - 3z &= 3 + 2 \times 1 - 3 \times 2 &= -1 \\ x - 3y - z &= 3 - 3 \times 1 - 2 &= -2 \end{cases}$$