$$(No.12)$$

1.

(1) $\displaystyle\int \frac{x}{\sqrt{9+x^2}}dx$

$$9+x^2=t \qquad\qquad \frac{dt}{dx}=2x \qquad \frac{1}{2}dt=xdx$$

$$\int \frac{1}{\sqrt{9+x^2}}xdx=\int\frac{1}{\sqrt{t}}\cdot\frac{1}{2}dt=\frac{1}{2}\int t^{-\frac{1}{2}}dt=\frac{1}{2}\left(\frac{1}{-\frac{1}{2}+1}t^{-\frac{1}{2}+1}\right)=\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}t^{\frac{1}{2}}=\sqrt{t}=\sqrt{9+x^2}$$

(2) $\displaystyle\int \frac{(\log x)^3}{x}dx$

$$t=\log x \qquad\qquad \frac{dt}{dx}=\frac{1}{x} \qquad dt=\frac{1}{x}dx$$

$$\int \frac{(\log x)^3}{x}dx=\int(\log x)^3\cdot\frac{1}{x}dx=\int t^3 dt=\frac{1}{4}t^4=\frac{1}{4}(\log x)^4$$

2.

(1) $\displaystyle\int_0^1 \sqrt{3x+1}\,dx$

$$3x+1=t \qquad\qquad \frac{dt}{dx}=3 \qquad \frac{1}{3}dt=dx$$

x	0	$\to$	1
t	1	$\to$	4

$$\int_0^1 \sqrt{3x+1}\,dx=\int_1^4 \sqrt{t}\cdot\frac{1}{3}dt=\frac{1}{3}\int_1^4 t^{\frac{1}{2}}dt=\frac{1}{3}\left[\frac{1}{\frac{1}{2}+1}t^{\frac{1}{2}+1}\right]_1^4$$

$$=\frac{1}{3}\left[\frac{1}{\frac{3}{2}}t^{\frac{3}{2}}\right]_1^4=\frac{1}{3}\left[\frac{2}{3}\sqrt{t^3}\right]_1^4=\frac{2}{9}\left[t\sqrt{t}\right]_1^4=\frac{2}{9}(4\sqrt{4}-1\sqrt{1})=\frac{2}{9}(4\cdot2-1\cdot1)=\frac{14}{9}$$

(2) $\displaystyle\int_0^{\log 3} e^x(e^x+2)^2 dx$

$$e^x+2=t$$

$$\frac{dt}{dx}=e^x \qquad dt=e^x dx$$

x	0	$\to$	$\log 3$
t	3	$\to$	5

$x=0$
$\quad t=e^0+2=1+2=3$
$x=\log 3$
$\quad t=e^{\log 3}+2=3+2=5$
$\{\quad e^{\log_e 3}=p$
$\quad\quad \log_e 3=\log_e p \qquad p=3\}$

$$\int_0^{\log 3} e^x(e^x+2)^2 dx=\int_3^5 t^2 dt=\left[\frac{1}{3}t^3\right]_3^5$$

$$=\frac{1}{3}\left[t^3\right]_3^5=\frac{1}{3}(5^3-3^3)=\frac{98}{3}$$

3.

(1) a,b,c

$$\frac{1}{x(x+1)^2}=\frac{a}{x}+\frac{b}{x+1}+\frac{c}{(x+1)^2}$$

$$1=a(x+1)^2+bx(x+1)+cx$$

$$1=a(x^2+2x+1)+b(x^2+x)+cx$$

$$1=(a+b)x^2+(2a+b+c)x+a$$

$$\begin{cases} a+b=0 \\ 2a+b+c=0 \\ a=1 \end{cases}$$

$$a=1,\ b=-1,\ c=-1$$

(2) $\displaystyle\int \frac{1}{x(x+1)^2}dx$

$$\int \frac{1}{x(x+1)^2}dx=\int\left(\frac{1}{x}+\frac{-1}{x+1}+\frac{-1}{(x+1)^2}\right)dx=\int\left\{\frac{1}{x}-\frac{1}{x+1}-(x+1)^{-2}\right\}dx$$

$$=\log|x|-\log|x+1|-\frac{1}{-2+1}(x+1)^{-2+1}=\log\left|\frac{x}{x+1}\right|+(x+1)^{-1}=\log\left|\frac{x}{x+1}\right|+\frac{1}{x+1}$$