積分法 ≪基礎≫ 小テスト $(No.2)$ 解答例

1. 次の不定積分を求めよ。

(1) $\int x^5 dx$

(解) $\int x^5 dx = \frac{1}{5+1}x^{5+1} = \frac{1}{6}x^6$ 〃

(2) $\int \frac{1}{\sqrt{x}} dx$

(解) $\int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{\frac{1}{2}}} dx = \int x^{-\frac{1}{2}} dx = \frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1} = \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} = 2\sqrt{x}$ 〃

(3) $\int \frac{1}{\sqrt{9-x^2}} dx$

(解) $\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{3^2-x^2}} dx = \mathrm{Sin}^{-1}\frac{x}{3}$ 〃

(4) $\int \frac{1}{x^2+25} dx$

(解) $\int \frac{1}{x^2+25} dx = \int \frac{1}{x^2+5^2} dx = \frac{1}{5}\mathrm{Tan}^{-1}\frac{x}{5}$ 〃

(5) $\int \frac{1}{\sqrt{x^2+3}} dx$

(解) $\int \frac{1}{\sqrt{x^2+3}} dx = \log\left|x+\sqrt{x^2+3}\right|$ 〃

2. 次の不定積分を求めよ。

(1) $\int \frac{(x-3)^2}{x} dx$

(解) $\int \frac{(x-3)^2}{x} dx = \int \frac{x^2-6x+9}{x} dx = \int \left(x-6+\frac{9}{x}\right) dx$

$= \int x dx - \int 6 dx + 9\int \frac{1}{x} dx = \frac{1}{2}x^2 - 6x + 9\log|x|$ 〃

(2) $\int (8e^x + 4\sin x) dx$

(解) $\int (8e^x + 4\sin x) dx = 8\int e^x dx + 4\int \sin x dx = 8e^x + 4(-\cos x) = 8e^x - 4\cos x$ 〃

(3) $\int (3\cos x + \sec^2 x) dx$

(解) $\int (3\cos x + \sec^2 x) dx = 3\int \cos x dx + \int \sec^2 x dx = 3\sin x + \tan x$ 〃

(4) $\int \cot^2 x dx$

(解) 公式 $1+\cot^2 x = \mathrm{cosec}^{\,2} x$ より、$\cot^2 x = \mathrm{cosec}^{\,2} x - 1$ であるから

$\int \cot^2 x dx = \int (\mathrm{cosec}^{\,2} x - 1) dx = \int \mathrm{cosec}^{\,2} x dx - \int 1 dx = -\cot x - x$ 〃

≪参考≫公式 $\sin^2 x + \cos^2 x = 1$ の両辺を $\sin^2 x$ で割ると

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$ $\therefore 1 + \left(\frac{\cos x}{\sin x}\right)^2 = \left(\frac{1}{\sin x}\right)^2$ $\therefore 1 + \cot^2 x = \mathrm{cosec}^2 x$