積分法 ≪基礎≫ 小テスト $(No.3)$ 解答例

1. 次の定積分の値を求めよ。

(1) $\displaystyle\int_{-2}^{1} x^4 dx$

(解) $\displaystyle\int_{-2}^{1} x^4 dx = \left[\frac{1}{5}x^5\right]_{-2}^{1} = \frac{1^5}{5} - \frac{(-2)^5}{5} = \frac{1}{5} - \frac{-32}{5} = \frac{33}{5}$ 〃

(2) $\displaystyle\int_{0}^{\frac{\pi}{4}} \sin x dx$

(解) $\displaystyle\int_{0}^{\frac{\pi}{4}} \sin x dx = \Big[-\cos x\Big]_{0}^{\frac{\pi}{4}} = \left(-\cos\frac{\pi}{4}\right) - (-\cos 0)$

$\displaystyle= -\frac{1}{\sqrt{2}} - (-1) = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$ 〃

(3) $\displaystyle\int_{1}^{e^3} \frac{1}{x} dx$

(解) $\displaystyle\int_{1}^{e^3} \frac{1}{x} dx = \Big[\log|x|\Big]_{1}^{e^3} = \log e^3 - \log 1 = 3\log e - 0 = 3 \times 1 = 3$ 〃

2. 関数 e^{-4x} を微分せよ。その結果を用いて、$\displaystyle\int_{-1}^{0} e^{-4x} dx$ を求めよ。

(解) $\left(e^{-4x}\right)' = e^{-4x} \cdot (-4x)' = e^{-4x} \cdot (-4) = -4e^{-4x}$ 〃

よって、$\displaystyle\left(-\frac{1}{4}e^{-4x}\right)' = e^{-4x}$ であるから

$\displaystyle\int_{-1}^{0} e^{-4x} dx = \left[-\frac{1}{4}e^{-4x}\right]_{-1}^{0} = \left(-\frac{1}{4}e^{0}\right) - \left(-\frac{1}{4}e^{-4(-1)}\right) = -\frac{1}{4} + \frac{1}{4}e^4 = \frac{1}{4}(e^4 - 1)$ 〃

3. 次の定積分の値を求めよ。

(1) $\displaystyle\int_{1}^{2} \left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)^2 dx$

(解) $\displaystyle\int_{1}^{2} \left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)^2 dx = \int_{1}^{2} \left(x - 6 + \frac{9}{x}\right) dx = \int_{1}^{2} x dx - \int_{1}^{2} 6 dx + 9\int_{1}^{2} \frac{1}{x} dx$

$\displaystyle= \left[\frac{1}{2}x^2\right]_{1}^{2} - \Big[6x\Big]_{1}^{2} + 9\Big[\log|x|\Big]_{1}^{2} = \left(2 - \frac{1}{2}\right) - (12 - 6) + 9(\log 2 - \log 1)$

$\displaystyle= \frac{3}{2} - 6 + 9\log 2 = -\frac{9}{2} + 9\log 2 = 9\left(\log 2 - \frac{1}{2}\right)$ 〃

(2) $\displaystyle\int_{0}^{\frac{\pi}{3}} \tan^2 x dx$

(解) 公式 $1 + \tan^2 x = \sec^2 x$ より、$\tan^2 x = \sec^2 x - 1$ であるから

$\displaystyle\int_{0}^{\frac{\pi}{3}} \tan^2 x dx = \int_{0}^{\frac{\pi}{3}} (\sec^2 x - 1) dx = \Big[\tan x - x\Big]_{0}^{\frac{\pi}{3}}$

$\displaystyle= \left(\tan\frac{\pi}{3} - \frac{\pi}{3}\right) - (\tan 0 - 0) = \sqrt{3} - \frac{\pi}{3}$ 〃