積分の応用 ≪基礎≫ 小テスト $(No.4)$ 解答例

1. 次の定積分の値を求めよ。

(1) $\displaystyle\int_{-2}^{2}(x^5-4x^3+3x^2-7x+5)dx$

（解）$\displaystyle\int_{-2}^{2}(x^5-4x^3+3x^2-7x+5)dx=\int_{-2}^{2}(x^5-4x^3-7x)dx+\int_{-2}^{2}(3x^2+5)dx$

$\displaystyle=0+2\int_{0}^{2}(3x^2+5)dx=2\Big[x^3+5x\Big]_0^2=2\{(2^3+5\times2)-(0^3-5\times0)\}=36$ 〃

(2) $\displaystyle\int_{-\frac{1}{4}}^{\frac{1}{4}}(x-4)(x^2+4x+16)dx$

（解）$\displaystyle\int_{-\frac{1}{4}}^{\frac{1}{4}}(x-4)(x^2+4x+16)dx=\int_{-\frac{1}{4}}^{\frac{1}{4}}(x^3-64)dx=\int_{-\frac{1}{4}}^{\frac{1}{4}}x^3dx-\int_{-\frac{1}{4}}^{\frac{1}{4}}64dx$

$\displaystyle=0-2\int_{0}^{\frac{1}{4}}64dx=-2\Big[\,64x\,\Big]_0^{\frac{1}{4}}=-2\Big\{\,64\times\frac{1}{4}-64\times0\,\Big\}=-2(16-0)=-32$ 〃

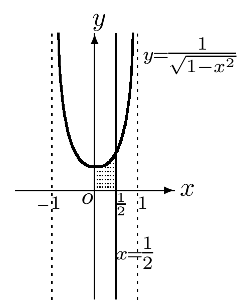
(3) $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}(\sin x-\cos x)dx$

（解）$\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}(\sin x-\cos x)dx=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\sin xdx-\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos xdx=0-2\int_{0}^{\frac{\pi}{6}}\cos xdx$

$\displaystyle=-2\Big[\sin x\Big]_0^{\frac{\pi}{6}}=-2\left(\sin\frac{\pi}{6}-\sin 0\right)=-2\left(\frac{1}{2}-0\right)=-1$ 〃

2. 次の図形の面積 S を求めよ。

(1) 曲線 $y=\dfrac{1}{\sqrt{1-x^2}}$ と両座標軸および $x=\dfrac{1}{2}$ で囲まれる図形。



（解）$\displaystyle S=\int_{0}^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}dx=\Big[\,\mathrm{Sin}^{-1}x\Big]_0^{\frac{1}{2}}$

$\displaystyle=\mathrm{Sin}^{-1}\frac{1}{2}-\mathrm{Sin}^{-1}0=\frac{\pi}{6}-0=\frac{\pi}{6}$ 〃

≪考え方≫ $\sin\frac{\pi}{6}=\frac{1}{2}\to\mathrm{Sin}^{-1}\frac{1}{2}=\frac{\pi}{6}$

$\sin 0=0\to\mathrm{Sin}^{-1}0=0$

（別解）$x=\sin\theta$ とおくと

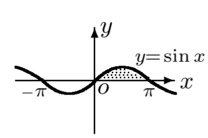
x	$0\to\frac{1}{2}$
θ	$0\to\frac{\pi}{6}$

$\dfrac{dx}{d\theta}=\cos\theta$ $\qquad\therefore dx=\cos\theta\ d\theta$

$\sqrt{1-x^2}=\sqrt{1-\sin^2\theta}=\sqrt{\cos^2\theta}=|\cos\theta|=\cos\theta$

$\displaystyle\int_{0}^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}dx=\int_{0}^{\frac{\pi}{6}}\frac{1}{\cos\theta}\cdot\cos\theta\ d\theta=\int_{0}^{\frac{\pi}{6}}1\ d\theta=\Big[\ \theta\ \Big]_0^{\frac{\pi}{6}}=\frac{\pi}{6}-0=\frac{\pi}{6}$ 〃

(2) 曲線 $y=\sin x$ $(0\leqq x\leqq\pi)$ と x 軸とで囲まれる図形。



（解）$\displaystyle S=\int_{0}^{\pi}\sin xdx=\Big[-\cos x\Big]_0^{\pi}$

$=(-\cos\pi)-(-\cos 0)=-\cos\pi+\cos 0$

$=-(-1)+1=2$ 〃