積分法 ≪基礎≫ 小テスト $(No.9)$ 解答例

1. 次の定積分の値を求めよ。

(1) $\displaystyle\int_1^3 \log x dx$ ≪考え方≫ $\displaystyle\int 1\, dx = x$ より、部分積分法を利用

(解) $\displaystyle\int_1^3 \log x dx = \int_1^3 1\cdot\log x dx = \Big[x\log x\Big]_1^3 - \int_1^3 x\cdot(\log x)'\,dx$

$\displaystyle = \Big[x\log x\Big]_1^3 - \int_1^3 x\cdot\frac{1}{x}dx = \Big[x\log x\Big]_1^3 - \int_1^3 1\,dx = \Big[x\log x\Big]_1^3 - \Big[x\Big]_1^3$

$= (3\cdot\log 3 - 1\cdot\log 1) - (3-1) = 3\log 3 - 1\times 0 - 2 = 3\log 3 - 2$ 〃

(2) $\displaystyle\int_0^{\frac{\pi}{6}} x\cos x dx$ ≪考え方≫ $\displaystyle\int \cos x dx = \sin x$ より、部分積分法を利用

(解) $\displaystyle\int_0^{\frac{\pi}{6}} x\cos x dx = \Big[x\sin x\Big]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}}(x)'\sin x dx = \Big[x\sin x\Big]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} 1\cdot\sin x dx$

$\displaystyle = \Big[x\sin x\Big]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}}\sin x dx = \Big[x\sin x\Big]_0^{\frac{\pi}{6}} - \Big[-\cos x\Big]_0^{\frac{\pi}{6}} = \Big[x\sin x\Big]_0^{\frac{\pi}{6}} + \Big[\cos x\Big]_0^{\frac{\pi}{6}}$

$\displaystyle = \left(\frac{\pi}{6}\sin\frac{\pi}{6} - 0\cdot\sin 0\right) + \left(\cos\frac{\pi}{6} - \cos 0\right) = \frac{\pi}{6}\cdot\frac{1}{2} - 0 + \frac{\sqrt{3}}{2} - 1 = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$ 〃

(3) $\displaystyle\int_0^{\sqrt{3}} \mathrm{Tan}^{-1}x dx$ ≪考え方≫ $\displaystyle\int 1 dx = x$ より、部分積分法を利用

(解) $\displaystyle\int_0^{\sqrt{3}} \mathrm{Tan}^{-1}x dx = \int_0^{\sqrt{3}} 1\cdot \mathrm{Tan}^{-1}x dx = \Big[x\ \mathrm{Tan}^{-1}x\Big]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} x(\ \mathrm{Tan}^{-1}x)'\,dx$

$\displaystyle = \Big[x\ \mathrm{Tan}^{-1}x\Big]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} x\cdot\frac{1}{1+x^2}dx = \Big[x\ \mathrm{Tan}^{-1}x\Big]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}}\frac{2x}{1+x^2}dx$

$\displaystyle = \Big[x\ \mathrm{Tan}^{-1}x\Big]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}}\frac{(1+x^2)'}{1+x^2}dx = \Big[x\ \mathrm{Tan}^{-1}x\Big]_0^{\sqrt{3}} - \frac{1}{2}\Big[\log|1+x^2|\Big]_0^{\sqrt{3}}$

$\displaystyle = (\sqrt{3}\ \mathrm{Tan}^{-1}\sqrt{3} - 0\cdot\ \mathrm{Tan}^{-1}0) - \frac{1}{2}(\log 4 - \log 1) = (\sqrt{3}\cdot\frac{\pi}{3} - 0\cdot 0) - \frac{1}{2}(\log 4 - 0)$

$\displaystyle = \frac{\sqrt{3}\pi}{3} - \frac{1}{2}\log 4 = \frac{\pi}{\sqrt{3}} - \frac{1}{2}\log 2^2 = \frac{\pi}{\sqrt{3}} - \frac{1}{2}\cdot 2\log 2 = \frac{\pi}{\sqrt{3}} - \log 2$ 〃

2. 次の不定積分を求めよ。

(1) $\displaystyle\int\frac{x^2}{x+3}dx$

(解) $\displaystyle\int\frac{x^2}{x+3}dx = \int\left(x - 3 + \frac{9}{x+3}\right)dx$

$\displaystyle = \int x dx - \int 3 dx + 9\int\frac{1}{x+3}dx = \frac{1}{2}x^2 - 3x + 9\log|x+3|$ 〃

≪計算≫

$$\begin{array}{r|l} & x-3 \\ x+3 & x^2 \\ & x^2+3x \\ & -3x \\ & -3x-9 \\ & 9 \end{array}$$

(2) $\displaystyle\int\frac{dx}{x^2-x-2}$

(解) $\displaystyle\int\frac{dx}{x^2-x-2} = \int\frac{1}{(x+1)(x-2)}dx = \frac{1}{3}\int\left(\frac{1}{x-2} - \frac{1}{x+1}\right)dx$

$\displaystyle = \frac{1}{3}(\ \log|x-2| - \log|x+1|\ ) = \frac{1}{3}\log\left|\frac{x-2}{x+1}\right|$ 〃

≪計算≫

$\frac{1}{(x+1)(x-2)} = \frac{a}{x+1} + \frac{b}{x-2}$

$1 = a(x-2) + b(x+1)$

$1 = (a+b)x + (-2a+b)$

$a+b = 0$, $-2a+b = 1$

$\therefore a = -\frac{1}{3}$, $b = \frac{1}{3}$

$\therefore \frac{1}{(x+1)(x-2)} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}}{x-2}$

$= \frac{1}{3}\left(\frac{1}{x-2} - \frac{1}{x+1}\right)$