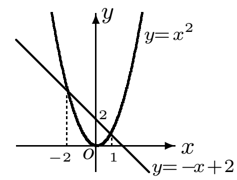
積分の応用 ≪基礎≫ 小テスト　解答例 (*No*.1)

1. 曲線 $y=x^2$ と直線 $y=-x+2$ に囲まれる図形の面積を求めよ。



（解）$S=\displaystyle\int_{-2}^{1}\left\{(-x+2)-x^2\right\}dx=\left[-\frac{1}{2}x^2+2x-\frac{1}{3}x^3\right]_{-2}^{1}$

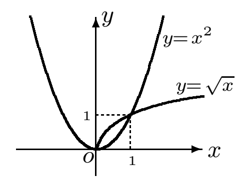
$=\left(-\frac{1}{2}\cdot1^2+2\cdot1-\frac{1}{3}\cdot1^3\right)-\left\{-\frac{1}{2}\cdot(-2)^2+2\cdot(-2)-\frac{1}{3}\cdot(-2)^3\right\}$

$=\left(-\frac{1}{2}+2-\frac{1}{3}\right)-\left(-2-4+\frac{8}{3}\right)=-\frac{1}{2}+8-\frac{9}{3}=-\frac{1}{2}+5=\frac{9}{2}$　〃

（別解）$S=\displaystyle\int_{-2}^{1}\left\{(-x+2)-x^2\right\}dx=-\int_{-2}^{1}(x^2+x-2)dx$

$=-\displaystyle\int_{-2}^{1}(x+2)(x-1)dx=-\left[-\frac{1}{6}\left\{1-(-2)\right\}^3\right]=\frac{1}{6}\cdot3^3=\frac{9}{2}$　〃

2. ２曲線 $y=x^2$，$y=\sqrt{x}$ で囲まれる図形の面積を求めよ。

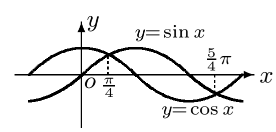


（解）$S=\displaystyle\int_{0}^{1}\left(\sqrt{x}-x^2\right)dx=\int_{0}^{1}\left(x^{\frac{1}{2}}-x^2\right)dx$

$=\left[\dfrac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1}-\dfrac{1}{2+1}x^{2+1}\right]_0^1=\left[\dfrac{1}{\frac{3}{2}}x^{\frac{3}{2}}-\dfrac{1}{3}x^3\right]_0^1$

$=\left[\frac{2}{3}\sqrt{x^3}-\frac{1}{3}x^3\right]_0^1=\left(\frac{2}{3}-\frac{1}{3}\right)-(0-0)=\frac{1}{3}$　〃

3. 区間 $[\,\frac{\pi}{2},\,\frac{3}{2}\pi\,]$ で、２曲線 $y=\sin x$，$y=\cos x$ で囲まれる図形の面積を求めよ。

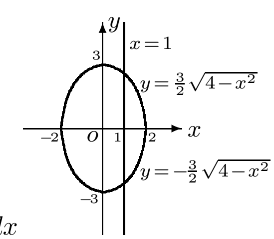


（解）$S=\displaystyle\int_{\frac{\pi}{4}}^{\frac{5}{4}\pi}(\sin x-\cos x)dx=\Big[-\cos x-\sin x\Big]_{\frac{\pi}{4}}^{\frac{5}{4}\pi}$

$=\left(-\cos\frac{5}{4}\pi-\sin\frac{5}{4}\pi\right)-\left(-\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\right)$

$=\left\{-\left(-\frac{1}{\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}\right)\right\}-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{4}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$　〃

4. 楕円 $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$ の内部で $x\geqq1$ の部分の図形の面積を求めよ。



（解）$\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$ を変形して、　$\dfrac{y^2}{9}=1-\dfrac{x^2}{4}$　$\therefore y^2=9\left(1-\dfrac{x^2}{4}\right)$

$\therefore y^2=\dfrac{9}{4}(4-x^2)$　$\therefore y=\pm\dfrac{3}{2}\sqrt{4-x^2}$

$S=\displaystyle\int_1^2\left\{\frac{3}{2}\sqrt{4-x^2}-\left(-\frac{3}{2}\sqrt{4-x^2}\right)\right\}dx=2\int_1^2\left(\frac{3}{2}\sqrt{4-x^2}\right)dx$

$=3\displaystyle\int_1^2\sqrt{2^2-x^2}dx=3\cdot\left[\frac{1}{2}\left(x\sqrt{2^2-x^2}+2^2\ \mathrm{Sin}^{-1}\frac{x}{2}\right)\right]_1^2$

$=\dfrac{3}{2}\left[x\sqrt{4-x^2}+4\ \mathrm{Sin}^{-1}\dfrac{x}{2}\right]_1^2=\dfrac{3}{2}\left\{\left(0+4\ \mathrm{Sin}^{-1}1\right)-\left(\sqrt{3}+4\ \mathrm{Sin}^{-1}\dfrac{1}{2}\right)\right\}$

$=\dfrac{3}{2}\left(4\cdot\dfrac{\pi}{2}-\sqrt{3}-4\cdot\dfrac{\pi}{6}\right)=3\pi-\dfrac{3\sqrt{3}}{2}-\pi=2\pi-\dfrac{3\sqrt{3}}{2}$　〃